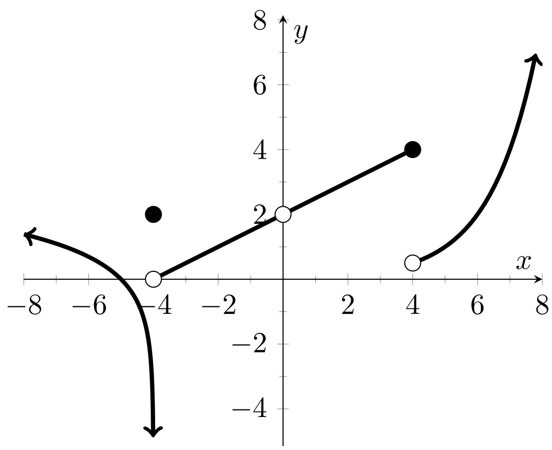
Math 251 Fall 2017 **Quiz #3, September 20**

Name: Solutions

There are 25 points possible on this quiz. This is a closed book quiz. Calculators and notes are not allowed. **Please show all of your work!** If you have any questions, please raise your hand.

Exercise 1. (5 pts.) Consider the function $f(x)$ with graph given below.



a.) List any values a where $\lim_{x\to a} f(x)$ fails to exist.

$-4, 4$

b.) List any values x where $f(x)$ fails to be continuous. Describe the type of discontinuity at each such value a.

-4 is an infinite discontinuity

0 is a removable discontinuity

4 is a jump discontinuity

Exercise 2. (4 pts.) Evaluate $\lim_{x\to 4} \frac{x^2-3x-4}{4-x}$.

$$\lim_{x\to 4} \frac{x^2-3x-4}{4-x} = \lim_{x\to 4} \frac{\cancel{(x-4)}(x+1)}{-\cancel{(x-4)}} = \lim_{x\to 4} -(x+1) = -5$$

Exercise 3. (4 pts.) Evaluate $\lim_{h\to 0} \frac{\frac{1}{2+h}-\frac{1}{2}}{h}$.

$$\lim_{h\to 0} \frac{\frac{1}{2+h}-\frac{1}{2}}{h} = \lim_{h\to 0} \frac{\frac{2-2-h}{2(2+h)}}{h} = \lim_{h\to 0} \frac{\frac{-h}{4+2h}}{h} = \lim_{h\to 0} \frac{-1}{4+2h} = \frac{-1}{4}.$$

Exercise 4. (5 pts.) Consider the function

$$f(x) = \begin{cases} x+1 & x<1 \\ 5 & x=1 \\ \frac{2}{x^2} & x>1 \end{cases}$$

a.) Evaluate $\lim\limits_{x\to 1} f(x)$.

① $\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} (x+1) = 2$

② $\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} \frac{2}{x^2} = \frac{\lim\limits_{x\to 1^+} 2}{\lim\limits_{x\to 1^+} x^2} = \frac{2}{1} = 2$

Limit ① = limit ②, so $\lim\limits_{x\to 1} f(x) = 2$.

b.) Explain why $f(x)$ fails to be continuous at $x = 1$.

$\lim\limits_{x\to 1} f(x) = 2 \neq 5 = f(1)$.

Exercise 5. (4 pts.) Using complete sentences, explain why the function $f(x) = x^2 - 2 - \cos x$ has a zero on the interval $[0, \pi]$.

Observe that $f(0) = 0^2 - 2 - 1 = -3 < 0$ and $f(\pi) = \pi^2 - 2 + 1 = \pi^2 - 1 > 0$. Also note that $f(x)$ is continuous on $[0, \pi]$, so by the Intermediate Value theorem there is a $0 < c < \pi$ such that $f(c) = 0$.

Exercise 6. (3 pts.) If $2x \le g(x) \le x^4 - x^2 + 2$ for all x, evaluate $\lim\limits_{x\to 1} g(x)$. Justify your answer.

Observe that $\lim\limits_{x\to 1} 2x = 2$ and $\lim\limits_{x\to 1} (x^4 - x^2 + 2) = 1 - 1 + 2 = 2$, so by the Squeeze theorem, $\lim\limits_{x\to 1} g(x) = 2$.